INTEGRAL AND DIFFERENTIAL OPERATORS AND THEIR APPLICATIONS

An International Conference in Honour of Professor Stefan Samko

June 30 - July 2, 2011
Aveiro, Portugal

http://idota2011.glocos.org/

with the support of FCT - Fundação para a Ciência e a Tecnologia (Portugal) through
The Scientific Community Support Program (FACC)
Version of June 22, 2011

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1 Introduction

The Center for Research and Development in Mathematics and Applications and the Department of Mathematics of University of Aveiro are pleased to invite you to IDOTA – an international conference in honour of Professor Stefan Samko on the occasion of his 70^{th} birthday – to be held in Aveiro, Portugal, from June 30 to July 2, 2011.

The topics of the conference include (but are not limited to):

- Integral and Differential Operators;
- Variable Exponent Analysis;
- Harmonic Analysis;
- Function Spaces;
- Nonlinear Analysis;
- Factorization Theory;
- Boundary Value Problems;
- and PDEs.

1.1 Plenary Speakers

- Martin Costabel (Université de Rennes 1, France)
- Roland Duduchava (Ivane Javakhishvili State University of Tbilisi, Georgia)
- George Hsiao (University of Delaware, USA)
- Vakhtang Kokilashvili (Georgian Academy of Sciences, Georgia)
- Carlos Pérez (University of Seville, Spain)
- Lars-Erik Persson (Luleå University of Technology, Sweden)
- António Ferreira dos Santos (Instituto Superior Técnico, Portugal)
- Ilya Spitkovsky (College of William and Mary, USA)
- Vladimir Rabinovich (National Polytechnic Institute, Mexico)
- Wolfgang Wendland (University of Stuttgart, Germany)

1.2 Scientific Committee

- Ravi P. Agarwal (Melbourne, USA)
- Luís Castro (Aveiro, Portugal)
- Martin Costabel (Rennes, France)
- Roland Duduchava (Tbilisi, Georgia)

- António Ferreira dos Santos (Lisbon, Portugal)
- Paulo Jorge Ferreira (Aveiro, Portugal)
- George Hsiao (Delaware, USA)
- Vakhtang Kokilashvili (Tbilisi, Georgia)
- David Natroshvili (Tbilisi, Georgia)
- Nikolaos Papageorgiou (Athens, Greece)
- Carlos Pérez (Sevilla, Spain)
- Lars-Erik Persson (Luleå, Sweden)
- Gueorgui Smirnov (Braga, Portugal)
- Frank-Olme Speck (Lisbon, Portugal)
- Ilya Spitkovsky (Williamsburg, USA)
- Saburou Saitoh (Aveiro, Portugal)
- Vasile Staicu (Aveiro, Portugal)
- Ioannis Stratis (Athens, Greece)
- José Miguel Urbano (Coimbra, Portugal)
- Vladimir Rabinovich (Mexico City, Mexico)
- Wolfgang L. Wendland (Stuttgart, Germany)

1.3 Organizing Committee

- Alexandre Almeida
- António Caetano
- Luís Castro (Chairman)
- Ana Paula Nolasco
- Humberto Rafeiro
- Eugénio Rocha
- Manuela Rodrigues
- Sandrina Santos
- Frank-Olme Speck

1.4 Conference Contact

Person: Prof. Luís Filipe Castro

E-mail: idota2011@glocos.org

2 Presentation titles and abstracts

The presentations include:

- **10 Invited Talks** of one-hour length (where 50 minutes are for the exposition and 10 minutes are for possible comments or questions):
  M. Costabel - Rennes; R. Duduchava - Tbilisi; G. Hsiao - Delaware; V. Kokilashvili - Tbilisi; C. Pérez - Seville; L.-E. Persson - Luleå; A. F. dos Santos - Lisbon; I. Spitkovsky - Williamsburg; V. Rabinovich - Mexico City; W. Wendland - Stuttgart.
- **2 Talks** to be included in the *Conference Opening Ceremony*: F.-O. Speck - Lisbon; S. Rogosin - Minsk.
- **63 Contributed Talks** of 30-minutes length (where 25 minutes are for the exposition and 5 minutes are for possible comments or questions).

List of Presentations
(ordered by title)

A BOUNDARY INTEGRAL APPROACH TO UNILATERAL CONTACT WITHOUT AND WITH TRESCA FRICTION IN HEMITROPIC ELASTICITY

Joachim Gwinner

The contribution is based on recent joint work with A. Gachechiladze, R. Gachechiladze, and D. Natroshvili [1,2,3].

Elastic materials may exhibit hemitropic behaviour on the atomic scale, as in quartz and in biological molecules, as well as on a large scale, as in composites with helical or screw–shaped inclusions.

In contrast to classical linear elasticity [4,5] the mechanical theory of hemitropic materials is formulated in terms of the displacement vector and an independent microrotation vector to describe intrinsic rotations of the particles. This gives rise to a generalized stress operator as a 6×6 matrix differential operator. Thus as traces at the boundary we obtain a force stress vector and a couple stress vector. Unilateral contact leads as usual to inequality constrains and complementarity condition for the displacement vector and the force stress vector. However, we have to pay special attention to the fact that the force stress and couple stress vectors depend on the displacement and microrotation vectors.

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A BRIEF REFLECTION OF STEFAN SAMKO'S LIFE AND WORK

Frank-Olme Speck

This is a special talk to be included in the opening ceremony.

A CLASS OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS

Magda Rebelo

In this talk we consider a class of nonlinear Volterra integral equations whose kernels have both diagonal and boundary singularities. The smoothness properties of the solutions are investigated. Their singular behaviour causes a reduction in the global convergence order of product integration and spline collocation methods on uniform meshes. In order to obtain a numerical solution with an optimal order of convergence we use collocation methods based on graded meshes with an appropriate grading exponent. Several numerical examples are presented and discussed.

A SINGULARLY PERTURBED DIRICHLET PROBLEM FOR THE POISSON EQUATION IN A PERIODICALLY PERFORATED DOMAIN

Paolo Musolino

This talk is devoted to the analysis of a singularly perturbed Dirichlet problem for the Poisson equation in a periodically perforated domain. We consider a sufficiently regular bounded open connected subset Ω of $\mathbb{R}^n$ such that $0 \in \Omega$ and such that the complement of the closure of Ω is connected. Then we choose a point $p \in]0,1[^n$. If ϵ is a small positive real number, we introduce the periodically perforated domain $S(\epsilon)^-$ obtained by removing from $\mathbb{R}^n$ the closure of the set $\cup_{z\in\mathbb{Z}^n}(p+\epsilon\Omega+z)$. For each positive and small ϵ, we consider a Dirichlet problem for the Poisson equation in the set $S(\epsilon)^-$. Namely, we consider a Dirichlet condition on the boundary of the set $p+\epsilon\Omega$, together with a periodicity condition. Then we show real analytic continuation properties of the solution as a functional of ϵ, of the

Dirichlet datum on $p + \epsilon\partial\Omega$, and of the Poisson datum, around a degenerate triple with $\epsilon = 0$.

This talk is based on joint work with Massimo Lanza de Cristoforis, University of Padova.

APPROXIMATION BY SOME EXTREMAL POLYNOMIALS IN CLOSED DOMAINS WITH MULTISMOOTH BOUNDARIES.

Burçin Oktay

Let G be a domain bounded by a smooth Jordan curve L. We define the subclass of the domains with a smooth boundary and in this subclass investigate the error

$$\| \phi_0 - \pi_n \|_{\overline{G}} := \max\{| \phi_0(z) - \pi_n(z) | : z \in \overline{G}\},$$

where ϕ_0 is the Riemann conformal mapping of the domain G onto unit disk under the conditions $\phi_0(z_0) = 0, \phi_0'(z_0) = 1$, for the fixed point $z_0 \in G$, and $\pi_n(z)$ is Bieberbach polynomials that minimize the norm

$$\|\phi_0' - p_n'\|_{L^2(G)} := \int_G |\phi_0' - p_n'|^2 \, dA(z),$$

in the class of algebraic polynomials p_n of degree at most n, satisfying $p_n(z_0) = 0$, $p_n'(z_0) = 1$.

ASYMPTOTICS OF THE EIGENVECTORS OF LARGE TOEPLITZ-HESSENBERG MATRICES GENERATED BY SYMBOLS WITH A POWER SINGULARITY

Egor Maximenko

We consider Hessenberg-Toeplitz matrices generated by symbols with one power singularity: $a(t) = t^{-1}(1-t)^{\alpha} f(t)$, where f is a function analytic and not vanishing in some neighborhood of the closed unit disk. Asymptotic formulas for the eigenvectors of these matrices are obtained as the order of the matrix goes to infinity. Our results confirm and adjust the hypothesis stated by H. Day, Z. Geary, and L. P. Kadanoff in 2009.

The talk is based on joint work with M. Bogoya, A. Böttcher, and S. Grudsky.

ATOMIC DECOMPOSITION OF BESOV SPACES OF VARIABLES SMOOTHNESS AND INTEGRABILITY

Drihem Douadi

The aim of this talk is twofold. First we characterize the Besov spaces of variables smoothness and integrability by so called local means. Secondly we use these results to present the atomic decomposition of these function spaces.

BOUNDARY VALUE PROBLEM AND LOCALIZATION

Roland Duduchava

Localization is a powerful tool in the investigation of the Fredholm properties of a boundary value problem for a partial differential equation in a domain with the smooth or piecewise-smooth boundary. It provides a better insight into the role of the Shapiro-Lopatinsky condition and, in combination with the uniqueness result and the index theorem, allows to prove the unique solvability of the boundary value problem.

The talk is based on the papers [1-3].

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BOUNDARY VALUE PROBLEMS ON GRATINGS WITH DIFFERENT SPACING WIDTHS

Ana Moura Santos

The present work deals with boundary value problems for the Helmholtz equation issued from wave diffraction by periodic gratings with different spacing widths. We briefly describe the formulation of the boundary value problems as convolution operators acting on Bessel potential periodic spaces and the equivalence to Toeplitz operators acting on spaces of matrix functions defined on composed contours. The corresponding Fourier symbols are represented as a sum of a matrix function not depending on the wave number and a symbol associated with a compact operator. Finally we analyze the Fredholm properties of the operators under consideration and get an approximate solution for small wave numbers for particular cases of boundary value problems.

This is a joint work with Amélia Bastos.

BOUNDEDNESS OF LINEAR OPERATORS IN WEIGHTED GRAND MORREY SPACES

Salaudin Umarkhadzhiev

Let $1 < p < \infty$, $\Omega \subseteq \mathbb{R}^n$ be an open set (which may be unbounded) and $\rho(x)$ a weight on Ω; $\theta > 0$, $0 \leq \lambda < 1$. By $M_\alpha^{p),\theta,\lambda}(\Omega, \rho)$ we denote the weighted grand Morrey space of functions $f : \Omega \to \mathbb{R}$ for which the following norm

$$\|f\|_{M_\alpha^{p),\theta,\lambda}(\Omega,\rho)} = \sup_{0<\varepsilon<p-1} \sup_{x\in\Omega, r>0} \left\{ \frac{\varepsilon^\theta}{(\mu B(x,r))^\lambda} \int\limits_{B(x,r)} |f(y)|^{p-\varepsilon} \rho(y) \langle y \rangle^{-\alpha\varepsilon} dy \right\}^{\frac{1}{p-\varepsilon}},$$

is finite, where $\alpha := \inf \left\{ \alpha > 0 : \ \sup_{x\in\Omega, r>0} (\mu B(x,r))^{-\lambda} \int_{B(x,r)} \rho(y) \langle y \rangle^{-\alpha p} dy < \infty \right\}$.

A class $W_p = W_p(\Omega), p \in (1, \infty)$ of weights on Ω will be called *allowable*, if it possesses properties

1. $w \in W_p \implies w \in W_{p-\varepsilon}$ for some $\varepsilon > 0$,
2. $w \in W_p \implies w^{1+\varepsilon} \in W_p$ for some $\varepsilon > 0$,
3. $w_1, w_2 \in W_p \implies w_1^t w_2^{1-t} \in W_p$ for every $t \in [0, 1]$.

As is well known, the Muckenhoupt class $A_p, 1 < p < \infty$ is allowable.

Let $W_p = W_p(\Omega)$ be a class of weights on Ω, depending on the parameter $p \in [1, \infty)$. We say that a linear operator T *belongs to the class* $\mathcal{B}(\Omega, W_p)$, if it is bounded in the space $L^p(\Omega, \rho)$ for every $\rho \in W_p$.

Theorem. *Let $\Omega \subseteq \mathbb{R}^n$ be an open set, $1 < p < \infty$ and W_p an allowable class of weights. If*

$$T \in \mathcal{B}(L^p(\Omega), W_p) \cap \mathcal{B}(L^{p-\varepsilon_0}(\Omega), W_{p-\varepsilon_0})$$

for some $\varepsilon_0 \in (0, p-1)$, then T is also bounded in the weighted grand Morrey space $M^{p),\theta,\lambda}(\Omega, w)$, where $w \in W_p$.

BOUNDEDNESS OF THE ANISOTROPIC RIESZ POTENTIAL IN ANISOTROPIC LOCAL MORREY-TYPE SPACES

Ali Akbulut

Let $d = (d_1, \dots, d_n)$, $d_i \geq 1$, $i = 1, \dots, n$, $|d| = d_1 + \dots + d_n$. In this talk, we consider general local and global anisotropic Morrey-type spaces $LM_{p\theta,w,d}$ and $GM_{p\theta,w,d}$ as in [1,2]. We study the boundedness of the anisotropic Riesz potential I^d_α, $0 < \alpha < |d|$ from $LM_{p_1\theta_1,w_1,d}$ to $LM_{p_2\theta_2,w_2,d}$ and from $GM_{p_1\theta_1,w_1,d}$ to $GM_{p_2\theta_2,w_2,d}$ for all admissible values of α, not necessarily $\alpha = |d| \left(\frac{1}{p_1} - \frac{1}{p_2}\right)$. We also consider separately the case in which $LM_{p_1\theta_1,w_1,d}$ and $GM_{p_1\theta_1,w_1,d}$ are replaced by $L_{p_1} \equiv L_{p_1}(\mathbb{R}^n)$. Moreover, for some values of the parameters we obtain necessary and sufficient conditions for the operator I^d_α, $0 < \alpha < |d|$ to be bounded from $LM_{p_1\theta_1,w_1,d}$ to $LM_{p_2\theta_2,w_2,d}$. Also, for all admissible values of the parameters, we obtain necessary and sufficient conditions for the operator I^d_α, $0 < \alpha < |d|$ to be bounded from L_{p_1} to $LM_{p_2\theta_2,w_2,d}$ or to $GM_{p_2\theta_2,w_2,d}$.

The problem of boundedness of the anisotropic Riesz potential I^d_α, $0 < \alpha < |d|$ in local Morrey-type spaces is reduced to the problem of boundedness of the Hardy operator in weighted L_p-spaces on the cone of non-negative non-increasing functions. This allows obtaining sharp sufficient conditions for boundedness for all admissible values of the parameters, which, for a certain range of the parameters wider than known before, coincide with the necessary ones.

This is a joint work of Ali Akbulut, Vagif S. Guliyev and Sh.A. Muradova.

References

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COBURN CONJECTURE FOR POLYANALYTIC FOCK SPACES

Nelson Faustino

The well know conjecture of Coburn announced in [5] and proved by Lo and Engliš (cf. [9, 6]) states that any Gabor-Daubechies operator with window ψ and symbol $a(x, \omega)$ quantised on the phase space by a Berezin-Toeplitz operator with window Ψ and symbol $\sigma(z, \overline{z})$ coincides with the Toeplitz operator with symbol $D\sigma(z, \overline{z})$ for some polynomial differential operator D.

The main goal of this talk will be around of the extension of such conjecture to polyanalytic Fock spaces using standard methods of quantisation, namely Berezin (cf. [4]) and Weyl quantisation (cf. [7]), showing that in a general case that D can also belong to a certain class of Gelfand-Shilov spaces (cf. [8]).

While the generation is almost mimetic for the true polyanalytic Fock spaces/generalized Bargmann spaces (cf. [1, 10]), the Gabor analysis framework for vector-valued windows (cf. [2, 3]) provides a meaningful extension of this conjecture to polyanalytic Fock spaces.

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COMMUTATIVE ALGEBRAS OF TOEPLITZ OPERATORS IN ACTION

Nikolai Vasilevski

We will discuss a quite unexpected phenomenon in the theory of Toeplitz operators on the Bergman space: the existence of a reach family of commutative C*-algebras generated by Toeplitz operators with non-trivial symbols. As it tuns out the smoothness properties of symbols do not play any role in the commutativity, the symbols can be merely measurable. Everything is governed here by the geometry of the underlying manifold, the hyperbolic geometry of the unit disk. We mention as well that the complete characterization of these commutative C*-algebras of Toeplitz operators requires the Berezin quantization procedure.

These commutative algebras come with a powerful research tool, the spectral type representation for the operators under study. This permit us to answer to many important questions in the area.

COMPLETENESS OF A SYSTEM OF ROOT FUNCTIONS FOR VARIOUS BOUNDARY VALUE PROBLEMS

Yakov Yakubov

We consider boundary value problems for elliptic differential operator equations with the spectral parameter and operator-boundary conditions containing the parameter of the same order as the equation or containing an unbounded operator. An isomorphism and the corresponding estimate of a solution (with respect to the space variable and the parameter) are obtained. Then, discreteness of the spectrum and completeness of a system of root functions of the corresponding homogeneous problem are established. Finally, an application is given of the abstract results obtained on the isomorphism and the completeness to boundary value problems for elliptic differential equations with a parameter in non-smooth domains.

CONTOUR INTEGRALS, PLEMELJ-PRIVALOV THEOREM ON NON-RECTIFIABLE JORDAN CURVES AND APPLICATIONS

Yevgeniy Guseynov

In this talk we will discuss the famous Plemelj-Privalov theorem on invariance of the Hölder classes H_a, with respect to a one-dimensional singular integral with Cauchy kernel on closed Jordan curves. V. V. Salaev posed the problem of describing the class P_a of all closed rectifiable Jordan curves on which the Plemelj-Privalov theorem is valid. Salaev's problem was solved in a paper by Salaev, Guseinov, and Seifullaev, where a condition completely characterizing the class P_a is given in terms of the planar measure of boundary strips of sets constructed from the curve. Here we present a definition of contour integral for closed non-rectifiable Jordan curves and Plemelj-Privalov theorem for such curves.

This is a joint work with Roustam Seif.

CONVOLUTION TYPE OPERATORS ON WEIGHTED LEBESGUE SPACES

Yuri Karlovich

Let $\mathfrak{B}_{p,w}$ denote the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$ where $1 < p < \infty$ and w is a Muckenhoupt weight. We study the Banach subalgebra $\mathfrak{A}_{p,w}$ of $\mathfrak{B}_{p,w}$ generated by all convolution type operators of the form

$a\mathcal{F}^{-1}b\mathcal{F}$ where $\mathcal{F}$ is the Fourier transform, the functions $a, b \in L^\infty(\mathbb{R})$ admit piecewise slowly oscillating discontinuities on $\mathbb{R} \cup \{\infty\}$ and b is a Fourier multiplier on $L^p(\mathbb{R}, w)$. Applying results on commutators of pseudodifferential operators with non-regular symbols and the Allan-Douglas local principle, we construct a Fredholm symbol calculus and obtain a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ in terms of their Fredholm symbols. An index formula for operators $A \in \mathfrak{A}_{p,w}$ is also established.

The talk is based on a joint work with I. Loreto Hernández.

DIRICHLET PROBLEM FOR ELLIPTIC EQUATIONS WITH VMO COEFFICIENTS IN GENERALIZED MORREY SPACES

Lubomira Softova

We consider the Dirichlet problem in bounded $C^{1,1}$-domain $\Omega \subset \mathbb{R}^n$ for the uniformly elliptic equation $\mathcal{L}u := \sum_{ij=1}^n a^{ij} D_{ij} u = f$ with VMO principal coefficients. The problem is uniquely solvable in $W^{2,p} \cap W_0^{1,p}(\Omega)$ for all $p \in (1, \infty)$. Our aim is to show that for every f belonging to the generalized Morrey space $L^{p,\omega}(\Omega)$, $p \in (1, \infty)$, $\omega : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+$ the operator $\mathcal{L} : W^{2,p,\omega} \cap W_0^{1,p}(\Omega) \to L^{p,\omega}(\Omega)$ is bijective and the estimate $\|D^2 u\|_{L^{p,\omega}(\Omega)} \leq C\|f\|_{L^{p,\omega}(\Omega)}$ holds.

DISCRETE FUNCTION THEORIES IN SEVERAL DIMENSIONS

Uwe Kaehler

In recent years one can observe an increasing interest in obtaining discrete counterparts for various continuous structures, in particular a discrete equivalent to continuous function theory. This is not only driven by the idea of creating numerical algorithms for different continuous methods of studying partial differential equations, but also for true discrete purposes, as can be seen, among others, by recent results of S. Smirnov which connects complex discrete function theory with problems in probability and statistical physics. While such ideas are very much developed in the complex case the higher-dimensional case is yet to be fully understood. This is mainly due to two facts. The first point is that while discrete complex analysis is under (more or less) continuous development since the 1940's discrete function theories started effectively only in the eighties and nineties with the construction of discrete Dirac operators either for creating a numerical counterpart to the method of Gürlebeck and Sprößig for the study of partial differential equations or for different problems in physics, e.g. Vaz, Forgy/Schreiber, Faustino/K./Sommen or the construction of a discrete equivalent of several complex variables in term of holomorphic functions on bricks by Bobenko/Mercat/Suris. The second point is that while continuous function theories in higher dimensions have a canonical approach based on Weyl relations and their Lie algebra structure, this is not true in the discrete case due to the fact that one has in general two discrete difference operators in each coordinate. Quite recently a way out of this problem was suggested in terms of so-called Sommen-Weyl relations. In this talk we would like to start with a short overview of discrete function theories connected to discrete Dirac operators. We want to show how Sommen-Weyl relations turn out to be a good tool in constructing the necessary operators and give some applications.

DISCRETE INTEGRAL EQUATIONS AND A COLLOCATION METHOD TO SOLVE IT

Berenice Damasceno

This paper concerns the nonlinear system $Ty = Vu$ where V is a causal operator and y and u are considered on the space of the regulated functions from a closed subset of the real numbers into a general Banach space. It is known that if we use the Dushnik integral in general time scales, bounding regulated functions and its dual the space of all functions with finite semivariation, the operator V can be represented as an integral of the Volterra-Stieltjes type on time scales. The main result in the paper is the synthesis of a collocation method -in the sense of Ramm and Castro, for solving ill-posed problems – for the original equation one. An example for scattering solutions in an electrical circuit appearing in black-box identification theory is presented.

This is a joint work with Fabio Roberto Chavarette, Geraldo Nunes Silva and Luciano Barbanti.

EMBEDDINGS OF SOBOLEV-TYPE SPACES INTO GENERALIZED HÖLDER SPACES INVOLVING K-MODULUS OF SMOOTHNESS

Júlio Neves

We present a sharp estimate of k-modulus of smoothness of a function f such that the norm of its distributional gradient $|\nabla^k f|$ belongs locally to the Lorentz space $L^{n/k,1}(\mathbb{R}^n)$, $k \in \mathbb{N}$, $k < n$, and its reverse form. Results are applied to establish necessary and sufficient conditions for continuous embeddings of Sobolev-type spaces, modelled upon rearrangement invariant Banach function spaces $X(\mathbb{R}^n)$, into generalized Hölder spaces defined by means of the k-modulus of smoothness ($k \in \mathbb{N}$).

This is joint work with Amiran Gogatishvili, Susana Moura and Bohumír Opic.

EXISTENCE AND UNIFORM STABILITY OF SOLUTIONS FOR FRACTIONAL FUNCTIONAL DIFFERENTIAL EQUATIONS

Moulay Rchid Sidi Ammi

In this paper, based on a fixed point theorem in Banach algebras we prove an existence result for fractional functional differential equations. Further, uniform stability of solutions is established. In the end an illustrative example is given.

EXISTENCE AND UNIQUENESS OF SOLUTIONS OF THE MULTI-TERM SEQUENTIAL FRACTIONAL DIFFERENTIAL EQUATION

Malgorzata Klimek

We consider a multi-term nonlinear sequential differential equation of fractional order, containing Caputo derivatives of arbitrary order. We assume that the nonlinear term does not depend on derivatives and that the sequential derivatives are given as $D^{\alpha_j} := {}^cD_{0+}^{\alpha_{j-1}} \, D^{\alpha_j - \alpha_{j-1}}$. Then we transform the considered equation into the corresponding fractional integral one, including stationary function of fractional derivative. It is reformulated as an equivalent fixed point condition on the space of functions continuous in an arbitrary bounded interval [0,b]. We observe that each stationary function of the highest order derivative yields a respective mapping on the $C[0, b]$ space. To prove the existence and uniqueness of solution, the contraction principle is applied and a one-parameter class of equivalent metrics on the space of continuous functions. The continuous solution exists in an arbitrary bounded interval and is given explicitly as a limit of iterations of the mapping generated by the stationary function. In two cases, the initial value problem is formulated and solved.

EXISTENCE OF SOLUTIONS FOR A CLASS OF SINGULAR NONLINEAR ELLIPTIC EQUATIONS IN LORENTZ SPACES

Kelly Murillo

In this talk, we discuss the existence of solutions for the Dirichlet nonlinear elliptic problems

$$\begin{cases} -div(q(x, u(x), \nabla u(x))) + a(x)u(x) & = f(x) \quad \text{in} \quad \Omega, \\ u(x) & = 0 \qquad \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $Q(u) = div(q(x, u(x), \nabla u(x)))$ is a Leray-Lions type operator, the function $a \in L^{\infty}_{loc}(\Omega)$ is such that $a(x) \geq 0$ for all $x \in \Omega$, and f is a function in a Lorentz space $L^{q,q_1}(\Omega)$. Note that the function a may have singularities on the boundary of Ω and that p-Laplacian and generalized curvature operators

are of Leray-Lions type. Using a generalization of the Lax-Milgram theorem, we show the existence of a solution $u \in W_0^{1,p}(\Omega) \cap L^{r,s}(\Omega)$ to Eq.(1) and an *a priori* estimate for the solution u with respect to the norm of f, for suitable values p, q, q_1, r and s.

This is a joint work with Lirong Huang and Eugénio Rocha.

EXTREMAL FUNCTIONS OF AN INEQUALITY ON HEISENBERG GROUP

Jianqing Chen

Using the variational method, we prove that the smallest positive constant in an interpolation inequality on Heisenberg group can be achieved. This constant was characterized by a minimal action solution of an elliptic equation on Heisenberg group. The main idea is to study a suitable minimization problem and establish several delicate estimates. We also use this idea to simplify a proof on the existence of extremal functions of a Sobolev inequality on Heisenberg group.

This is a joint work with Eugénio Rocha.

FOURIER MULTIPLIER THEOREMS FOR BESOV AND TRIEBEL-LIZORKIN SPACES WITH VARIABLE EXPONENTS

Takahiro Noi

Let $p(\cdot), q(\cdot)$ and $s(\cdot)$ are $C^{\log}(\mathbb{R}^n)$ functions with $0 < p(\cdot), q(\cdot) < \infty$. Let $v \in \mathbb{N}$ and

$$||m||_v = \sup_{|u| \le v} \sup_{x \in \mathbb{R}^n} (1 + |x|^2)^{\frac{|u|}{2}} |D^u m(x)|$$

for an infinity differentiable function $m(\cdot)$. Then I will show that the following Fourier multiplier theorem for Besov and Triebel-Lizorkin spaces with variable exponents holds: If the natural number v is sufficiently large, then there exists a positive number c such that

$$||\mathcal{F}^{-1} m \mathcal{F} f||_{F^{s(\cdot)}_{p(\cdot),q(\cdot)}} \le c ||m||_v ||f||_{F^{s(\cdot)}_{p(\cdot),q(\cdot)}}$$

for all infinitely differentiable functions $m(\cdot)$ and all $f \in F^{s(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$. Similary, if the natural number v is sufficiently large, then there exists a positive number c such that

$$||\mathcal{F}^{-1} m \mathcal{F} f||_{B^{s(\cdot)}_{p(\cdot),q(\cdot)}} \le c ||m||_v ||f||_{B^{s(\cdot)}_{p(\cdot),q(\cdot)}}$$

for all infinitely differentiable functions $m(\cdot)$ and all $f \in B^{s(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$.

FRACTIONAL CALCULUS OF VARIATIONS INVOLVING MULTIPLE INTEGRALS

Agnieszka B. Malinowska

The Fractional Calculus of Variations (FCV) unifies the calculus of variations and the fractional calculus, by inserting fractional derivatives into the variational integrals. This occurs naturally in many problems of physics or mechanics, in order to provide more accurate models of physical phenomena. For example, the fractional operators are non-local, therefore they are suitable for constructing models possessing memory effect. The FCV started in 1996 with the work of Riewe. He formulated the problem of the calculus of variations with fractional derivatives and obtained the respective Euler–Lagrange equation, combining both conservative and nonconservative cases. Nowadays the FCV is a subject under strong research. The purpose of the present work is to study problems of the fractional calculus of variations with multiple integrals. The fractional derivatives are defined in the sense of Caputo. We present necessary and sufficient optimality conditions for problems of the fractional calculus of variations with multiple integrals.

FRACTIONAL TWO-PARAMETER SCHRÖDINGER EQUATION

M. Manuela Rodrigues

This work is intended to investigate the multi-dimensional space-time fractional Schrödinger equation of the form

$$\left({}^C D^{\alpha}_{t_0^+} u\right)(t,x) = \frac{i\hbar}{2m}\left({}^C\nabla^{\beta} u\right)(t,x)$$

with $\hbar$ the Planck's constant divided by 2π, m is the mass and $u(t,x)$ is a wave function of the particle. Here ${}^C D^{\alpha}_{t_0^+}, {}^C\nabla^{\beta}$ are operators of the Caputo fractional derivatives, where $0 < \alpha \leq 1$ and $1 < \beta \leq 2$. The wave function is obtained using Laplace and Fourier transforms methods and a symbolic operational form of solutions in terms of the Mittag-Leffler functions is exhibited. It is presented an expression for the wave function and for the quantum mechanical probability density. Using Banach fixed point theorem, the existence and uniqueness of solutions is studied for this kind of fractional differential equations.

FUNCTION SPACES AND OPERATOR EXTENSIONS RELATED TO MAXWELL EQUATIONS

Martin Costabel

There is a peculiar phenomenon that has been observed in the theoretical and numerical treatment of time-harmonic Maxwell equations: It is possible and, in fact, quite natural to

treat the standard boundary value and eigenvalue problems in "wrong" function spaces and then find solutions that are different from the "true" physical solutions. This has been seen at least 20 years ago in the framework of regularized variational formulations on non-smooth domains, and it has been known to lead to wrong numerical solutions by the finite element method. There the Maxwell equations with perfect conductor boundary conditions can be formulated either in the space $H(curl)$ or in the space H^1; both formulations are equivalent to the original boundary value problem in a distributional sense; for both formulations one can show existence and uniqueness; but the two unique solutions are different. A similar situation exists for a regularized boundary integral equation formulation of the perfect conductor scattering problem on a smooth open surface. Quite recently, a similar situation has been detected for the volume integral operator describing scattering by a magnetically active body. In the talk, we describe these three cases and explain the origin of the problem. Whereas in the first two cases, the problem is due to the presence of corner singularities, in the last case the problem also appears for smooth domains. In all cases, the phenomenon is related to the non-density of smooth functions in the function spaces defined by the electromagnetic energy and the possibility to define different natural selfadjoint extensions of the differential or integral operators in question.

GENERALIZED FRACTIONAL INTEGRATION BY PARTS

Tatiana Odzijewicz

In this work we study three types of generalized fractional operators. We give a proper extension of fractional integration by parts formula, by considering integral operators with more general kernels. New results are obtained even in the particular case, when the generalized operators are reduced to the standard fractional derivatives and fractional integrals in the sense of Riemann-Liouville or Caputo.

This is a joint work with Agnieszka Malinowska, Delfim Torres.

INVERSE PROBLEMS FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS IN A HILBERT SPACE

Mahmoud El-Borai

This note is devoted to study an inverse Cauchy problem in a Hilbert space H for fractional abstract differential equations of the form;

$$\frac{d^\alpha u(t)}{dt^\alpha} \;=\; A\, u(t) \;+\; f(t)\; g(t) + F(t, W(t)),$$

with the initial condition $u(0) = u_0 \;\in\; H$ and the overdetermination condition:

$$(u(t), v) = w(t),$$

where (.,.) is the inner product in H, f is a real unknown function w is a given real function, u_0, v are given elements in H, g is a given abstract function with values in H, $0 < \alpha \leq 1$, u is unknown, and A is a linear closed operator defined on a dense subset of H, $W(t) = (B_1(t)u(t), ..., B_r(t)u(t)), \{B_i(t) : i = 1, ..., r, t \in J\}$ is a family of linear closed operators defined on dense sets $S_1, ..., S_r \supset S$ respectively in H into H, F is a given abstract function on $J \times H^r$ into H.
It is supposed that A generates a semigroup. An application is given to study an inverse problem in a suitable Sobolev space for general nonlinear fractional parabolic partial differential equations with unknown source functions.

KERNELS OF ASYMMETRIC TOEPLITZ OPERATORS AND APPLICATIONS TO ALMOST PERIODIC FACTORIZATION

Ilya Spitkovsky

Necessary and sufficient factorability conditions are established for some new classes of almost periodic matrix functions, and explicit factorization formulas are given whenever the factorization exists. The results are based on the connection between factorability and properties of solutions to a related homogeneous Riemann-Hilbert problem. These solutions, in their turn, are described completely, for the cases at hand, via introduction and consideration of certain asymmetric Toeplitz operators.

The talk is based on joint work with C. Câmara and Yuri Karlovich.

KONTOROVICH-LEBEDEV'S TRANSFORM IN AN ANALYSIS OF TIME-DEPENDENT SCHRÖDINGER EQUATION

Nelson Vieira

In this talk we introduce a notion of Schrödinger's kernel to the familiar Kontorovich-Lebedev transform. In order to control its singularity at infinity, we will need to implement the so-called regularization procedure. Hence we will obtain a sequence of regularized kernels which converge to the original kernel when a regularization parameter tends to zero. We study differential and semigroup properties of the regularized kernel and construct fundamental solutions of a regularized time-dependent Schrödinger's equation. The correspondent regularized integral transformation is presented. We also establish analogs of the classical Heisenberg inequality and uncertainty principle for this transformation. Finally, we examine a pointwise convergence of this family of regularized integral operators, when the regularization parameter tends to zero.

LOCALIZATION OF PSEUDODIFFERENTIAL OPERATORS IN THE LEBESGUE SPACES WITH VARIABLE EXPONENTS.

Vladimir Rabinovich

The talk is devoted to the problem of localization of pseudodifferential operators in the Lebesgue spaces with variable exponents $L^{p(x)}(R^n)$ and applications to the Fredholm theory of pseudodifferential operators in the Sobolev spaces connected with $L^{p(x)}(R^n)$, and boundary values problems for them on compact and noncompact manifolds.

LOCALIZED HARMONIC PARAMETRIX METHOD FOR SCALAR ELLIPTIC EQUATIONS WITH VARIABLE COEFFICIENTS

David Natroshvili

We develop *localized boundary-domain integral equation* (LBDIE) method for second order scalar elliptic partial differential equations with variable matrix coefficients. We consider the Dirichlet, Neumann and Robin type boundary value problems (BVP) and with the help of the *localized harmonic parametrix method* we reduce them to the LBDIE system. In our case, the localized harmonic parametrix is represented as the product of the fundamental solution of the Laplace operator by an appropriately chosen cut-off function supported in some neighbourhood of the origin.

Clearly, the kernels of the corresponding localized layer and volume potentials constructed by means of the *localized harmonic parametrix* are supported in some neighbourhood of the reference point and they do not solve the original differential equation with variable coefficients.

First we study mapping properties of localized potentials and the corresponding boundary operators which are applied to reduce the BVPs to the corresponding LBDIE systems. In spite of the fact that the localized potentials preserve almost all mapping properties of the classical non-localized ones, some unusual properties of the localized potentials appear which have no counterparts in the classical potential theory and which need special consideration and analysis.

Afterwards we establish the equivalence between the original boundary value problems and the corresponding LBDIE systems which plays a crucial role in our analysis.

Finally, we establish that the localized boundary domain integral operators (LBDIO) obtained belong to the Boutet de Monvel algebra of pseudo-differential operators. On the basis of the Vishik-Eskin theory we investigate Fredholm properties of the LBDIO and prove their invertibility in appropriate function spaces.

The LBDIE approach developed seems to be useful for construction of efficient LBDIE based numerical algorithms for the BVP solution.

LOW RANK APPROXIMATIONS FOR INTEGRAL OPERATOR WITH FIRST EXPONENTIAL INTEGRAL FUNCTION KERNEL

Ana Luísa Nunes

A hierarchical matrix, $\mathcal{H}$-matrix for brief, is an efficient data-sparse representation of a matrix resulting from the discretization of an integral operator, especially useful for large dimensional problems. It consists, mostly, on low-rank subblocks leading to low memory requirements as well as cheap computational costs. In this work, we discuss the use of the $\mathcal{H}$-matrix technique in the numerical solution of a large scale eigenvalue problem arising from a finite rank discretization of an integral operator. The opertor is of convolution type, it is defined through the first exponential-integral function and hence it is weakly singular. We develop analytical expressions for the approximate degenerate kernels and deduce error upper bounds for these approximations. Some computational results illustrating the effciency and robustness of the approach are presented.

This is a joint work with Paulo B. Vasconcelos, Mario Ahues.

MODELING OF THE FERROELECTRIC HYSTERESIS AS VARIATIONAL INEQUALITY

Anna-Margarete Saendig

Ferroelectric materials are characterized by interaction-effects of mechanical and electrical fields due to different polarization directions of the unit cells. The relations between polarisation and electrical field and mechanical strain and electrical field respectively can be described by hysteresis curves. Some models, which describe the ferroelectrical material behaviour, e.g. [2, 4], rely on concepts closed to elastoplasticity. We use these ideas and derive variational evolution inequalities analogously to elastoplasticity, see [1]. Based on these inequalities we formulate equivalent mathematical problems and get some existence results, see [3]. We think that the formulation of variational evolution inequalities is a good starting point for numerical methods similar to elastoplasticity.

This is a joint work with Michael Kutter.

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NEW TYPES OF SOLUTIONS OF NON-LINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

Mark Edelman

Using Riemann-Liouville and Caputo Fractional Standard Maps (FSM) and Fractional Zaslavsky Map (FZM) as examples, we demonstrate new types of solutions for non-linear fractional differential equations: attractors which overlap, trajectories which intersect, cascade of bifurcation type trajectories.

This is a joint work with Laura Anna Taieb.

ON A NONORTHOGONAL POLYNOMIAL SEQUENCE ASSOCIATED TO THE MACDONALD WEIGHT FUNCTION

Ana Loureiro

A polynomial sequence generated by powers of a modified Bessel operator will be under discussion. The integral representation along with the generating function will be revealed via the Kontorovich-Lebedev transform. After presenting an explicit expression for such polynomial sequence accomplished by means of the so-called modified Stirling numbers, the corresponding dual sequence will be thoroughly constructed. It turns out that the canonical element of this dual sequence can be represented through the Macdonald weight function (modified Bessel function of second kind). Finally, we make some considerations about the corresponding orthogonal polynomial sequence.

The talk is based on a joint work with S. Yakubovich.

ON A REACTION-DIFFUSION SYSTEM WITH NONLOCAL IN TIME REACTION TERMS AND ANOMALOUS DIFFUSION

Mokhtar Kirane

For a system of reaction diffusion equations coupled with nonlocal in time reaction terms, we show global existence for a certain range of the parameters, blowing-up solutions for the complementary range of parameters are shown relying on the test function method of Pohozaev. Moreover, the profile of the blowing-up solutions is found.

ON A STRUCTURE OF THE KERNEL OF SINGULAR INTEGRAL OPERATORS WITH LINEAR-FRACTIONAL SHIFT

Oleksandr Karelin

We denote the Cauchy singular integral operator along the real axis $\mathbb{R}$ by

$$(S_{\mathbb{R}}\varphi)(t) = \frac{1}{\pi i}\int\limits_{\mathbb{R}} \frac{\varphi(\tau)}{\tau - t}d\tau,$$

and the identity operator on $\mathbb{R}$ by $(I_{\mathbb{R}}\varphi)(t) = \varphi(t)$.

We consider a singular integral operator

$$B_{\mathbb{R}} = aI_{\mathbb{R}} + bQ_{\mathbb{R}} + cS_{\mathbb{R}} + dQ_{\mathbb{R}}S_{\mathbb{R}} : L_2(\mathbb{R}) \to L_2(\mathbb{R}),$$

where the coefficients a, b, c, d are bounded measurable functions,

$$(Q_{\mathbb{R}}\varphi)(x) = \frac{\sqrt{\delta^2 + \beta}}{x - \delta}\varphi[\alpha(x)], \quad \alpha(x) = \frac{\delta x + \beta}{x - \delta}, \quad x \in \mathbb{R},$$

and δ, β are real numbers such that $\delta^2 + \beta > 0$.

We study the structure of the kernel of singular integral operators with involution $B_{\mathbb{R}}$ by using operators equalities as the main tools [1].

This talk is based on a joint work with Anna Tarasenko.

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ON BMO ESTIMATES FOR THE P-LAPLACIAN

Lars Diening

We consider the p-Laplacian system

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = -\mathrm{div}(G).$$

We show that $G \in \mathrm{BMO}$ implies $A(\nabla u) := |\nabla u|^{p-2}\nabla u \in \mathrm{BMO}$. This is the border case of the nonlinear Calderon-Zygmund theory.

ON BOUNDARY-DOMAIN INTEGRAL EQUATIONS FOR VARIABLE-COEFFICIENT BVPS IN EXTERIOR DOMAINS

Sergey E. Mikhailov

Some direct systems of boundary-domain integral equations (BDIEs) associated with the mixed boundary value problems for scalar PDEs with variable coefficients in exterior domains are formulated and analyzed in the paper. The BDIE equivalence to the original boundary value problems and the invertibility of the corresponding boundary-domain integral operators are proved in weighted Sobolev (Beppo Levi type) function spaces suitable for infinite domains. This extends the results obtained in [1-3] for interior domains in Sobolev spaces without weights.

This is a joint work with Otar Chkadua and David Natroshvili.

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ON RIESZ MINIMAL ENERGY PROBLEMS

Wolfgang Wendland

In $\mathbb{R}^n$, $n \geq 2$, we study the constructive and numerical solution of minimizing the energy relative to the Riesz kernel $|x-y|^{\alpha-n}$, where $1 < \alpha < n$, for the Gauss variational problem, considered for finitely many compact, mutually disjoint, boundaryless $(n-1)$-dimensional Lipschitz manifolds Γ_ℓ, $\ell \in L$, each Γ_ℓ being charged with Borel measures with the sign $\alpha_\ell = \pm 1$ prescribed. We show that the Gauss variational problem over an affine cone of Borel measures can alternatively be formulated as a minimum problem over an affine cone of surface distributions belonging to the Sobolev–Slobodetski space $H^{-\varepsilon/2}(\Gamma)$, where $\varepsilon := \alpha - 1$ and $\Gamma := \bigcup_{\ell \in L} \Gamma_\ell$. This allows the application of simple layer boundary integral operators on Γ and, hence, a penalty approximation. A corresponding numerical method is based on the Galerkin–Bubnov discretization with piecewise constant boundary elements. For $n = 3$ and $\alpha = 2$, multipole approximation and in the case $1 < \alpha < 3 = n$ wavelet matrix compression is applied to sparsify the system matrix. To the discretized problem, a projected gradient method is applied. Numerical results are presented to illustrate the approach.

This is a lecture on joint work with H. Harbrecht, G. Of and N. Zorii.

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ON SOME BOUNDARY VALUE PROBLEMS FOR GENERALIZED ANALYTIC FUNCTIONS

Peter Berglez

For the solutions of the Bers-Vekua equation $Dw := w_{\bar{z}} - c(z, \bar{z})\,\bar{w} = 0$ defined in a domain $\mathbb{D} \subset \mathbb{C}$ we consider Riemann-Hilbert type boundary conditions.

In the case of the existence of certain differential operators with which all the solutions of $Dw = 0$ defined in $\mathbb{D}$ can be generated from a function f holomorphic in $\mathbb{D}$ the boundary value problem is reduced to a Goursat problem for f in essence. For certain classes of coefficients c and domains $\mathbb{D}$ we show how this problem can be solved explicitely.

For the poly-pseudoanalytic functions obeying the differential equation $D^k w = 0, k \in \mathbb{N}$, we investigate an appropriate boundary value problem and give the solution for particular cases also.

This is a joint work with Thi Tuyet Luong.

ON THE GUROV-RESHETNYAK AND MUCKENHOUPT CONDITIONS FOR WEIGHTS

Lauri Berkovits

We discuss the so-called Gurov-Reshetnyak classes in the context of doubling metric measure spaces. Their connection to Muckenhoutp's A_p classes and reverse Hölder classes RH_p are considered. We present several applications of Gurov-Reshetnyak classes to asymptotical behaviour of embeddings between the related weight classes A_p and RH_p.

ON THE TYPE OF CONVERGENCE IN ATOMIC REPRESENTATIONS

António Caetano

The type of convergence in atomic representations in Besov and Triebel–Lizorkin spaces is usually presented in the sense of the topology of the tempered distributions, occasionally with some remarks about the possibility of the convergence being valid in some Lebesgue spaces, if some conditions are met. Here we show that, apart from borderline situations, those representations indeed converge in the Besov or Triebel–Lizorkin spaces themselves. We also deal with a corresponding question for wavelet representations in a recently introduced class of generalized local Hardy spaces.

ON THE UNIFORM DECAY IN CAUCHY VISCOELASTIC PROBLEMS

Mohammad Kafini

In this paper we consider a linear Cauchy viscoelastic problem with an external source term. We show that, for compactly supported initial data and for an exponentially decaying relaxation function, the decay of the first energy of solution is polynomial. The finite-speed propagation is used to compensate for the lack of Poincare's inequality in the whole space.

OVERVIEW OF FRACTIONAL DIFFERENCE OPERATORS AND LINEAR SYSTEMS

Ewa Girejko

Fractional difference operators and linear systems are discussed. Similarly as in the continuous case there are two ways to introduce the fractional operators like a generalization of the corresponding derivatives/differences or integrals/sums. Considering differences of higher orders a generalization usually used in fractional control theory can be developed. Another way is to take the approach taken by Miller and Ross and later extended by Atici and Eloe. We give characterization of such operators and show relations among them. Going further we ask about differences between solutions and applications of linear fractional differences systems connected with presented operators.

This is a joint work with Dorota Mozyrska.

PARTNER POTENTIALS OF THE QUANTUM HARMONIC OSCILLATOR

Jonathan Fellows

We derive and describe a two parameter family of one dimensionsonal quantum potentials, the Schrödinger equation for which is exactly soluble because they are supersymmetric partner potentials of the standard quantum harmonic oscillator. The potentials we discuss are nearly isospectral to the harmonic oscillator except that the energy gap between their ground and first excited states is a tunable parameter. We discuss possible physical applications for these potentials and further applications of the methods we used in deriving them.

This is a joint work with Robert Smith.

POTENTIAL OPERATORS IN GENERALIZED WEIGHTED MORREY SPACES

Natasha Samko

We study the boundedness of Hardy operators in weighted generalized Morrey spaces defined by a quasi-monotone function $\varphi(r)$ and a quasi-monotone radial weight $w(|x|)$ (the off-diagonal case), and apply this result to a similar boundedness of potential-type Operators.

QUADRATIC AND SUBEXPONENTIAL DECAY ESTIMATES FOR COMMUTATORS OF SINGULAR INTEGRAL OPERATORS

Carlos Pérez

Commutators of singular integral operators with BMO functions were introduced in the seventies by Coifman-Rochberg and Weiss. They are very interesting operators for many reasons and their study became a classical topic in modern harmonic analysis. One reason of this great interest is due to the fact that they are more singular than the usual singular integral operators. This idea can be expressed in many ways. In this lecture we plan to give two more reasons showing this worst behavior. One of them is related to a sharp L2 weighted estimate with respect A2 weights but the novelty is that the bound in term of the A2 constant of the weight is quadratic and no better while in the case of singular integrals is simply linear. The second reason is due to the fact that there is an appropriate local sub-exponential decay which in the case of singular integrals is of exponential type instead.

The first part is a joint work with D. Chung and C. Pereyra and the second with C. Ortiz.

REAL ANALYTIC FAMILIES OF HARMONIC FUNCTION IN A DOMAIN WITH A SMALL HOLE

Matteo Dalla Riva

Let $n \geq 3$ be a natural number. Let Ω^i and Ω^o be open bounded connected subsets of the Euclidean n-dimensional space $\mathbb{R}^n$. We assume that there exists $\epsilon_0 > 0$ such that the closure of $\epsilon\Omega^i$ is contained in Ω^o for all $\epsilon \in]-\epsilon_0, \epsilon_0[$. Then, for a fixed $\epsilon \in]-\epsilon_0, \epsilon_0[\setminus\{0\}$ we consider a Dirichlet problem for the Laplace operator in the perforated domain $\Omega^o \setminus \epsilon\Omega^i$ with boundary values which depend on ϵ in a way which will be clarified. We denote by u_ϵ the corresponding solution. If p is a point of Ω^o and $p \neq 0$, then we know that under suitable regularity assumptions there exist $\epsilon_p > 0$ and a real analytic operator U_p from $]-\epsilon_p, \epsilon_p[$ to $\mathbb{R}$ such that $u_\epsilon(p) = U_p[\epsilon]$ for all $\epsilon \in]0, \epsilon_p[$. In particular, we know that equality $u_\epsilon(p) = U_p[\epsilon]$ holds for ϵ small and positive. Instead, both $u_\epsilon(p)$ and $U_p[\epsilon]$ are defined also for ϵ negative. Thus it is natural to ask what happens to the equality $u_\epsilon(p) = U_p[\epsilon]$ when ϵ is negative. By exploiting a general result on the continuation properties of some particular real analytic families of harmonic function in domains with a small hole, we answer to this question and we show that the answer is different if the dimension n is even or odd.

These results were obtained in collaboration with Dr. Paolo Musolino (Padova).

RECENT PROGRESS IN STUDYING BOUNDEDNESS OF CLASSICAL OPERATORS AND COMMUTATORS OF REAL ANALYSIS IN GENERAL MORREY-TYPE SPACES

Vagif Guliyev

Suppose that T represents a linear or a sublinear operator, which satisfies that for any $f \in L_1(\mathbb{R}^n)$ with compact support and $x \notin supp f$

$$|Tf(x)| \leq c_0 \int_{\mathbb{R}^n} \frac{|f(y)|}{|x-y|^n} dy, \tag{2}$$

where c_0 is independent of f and x. Similarly, we assume that T_α represents a linear or a sublinear operator, which satisfies that for any $f \in L_1(\mathbb{R}^n)$ with compact support and $x \notin supp f$

$$|T_\alpha f(x)| \leq c_1 \int_{\mathbb{R}^n} \frac{|f(y)|}{|x-y|^{n-\alpha}} dy \tag{3}$$

for some $\alpha \in (0, n)$, where c_1 is independent of f and x.

We point out that the condition (2) was first introduced by Soria and Weiss (1994). The conditions (2) and (3) are satisfied by many interesting operators in harmonic analysis, such as the Calderón–Zygmund operators, Carleson's maximal operators, Hardy–Littlewood maximal operators, C. Fefferman's singular multipliers, R. Fefferman's singular integrals, Ricci–Stein's oscillatory singular integrals, the Bochner–Riesz means and so on.

We prove the boundedness of the sublinear operators T satisfying condition (2) generated by Calderón-Zygmund operators from one generalized Morrey space M_{p,φ_1} to another M_{p,φ_2},

$1 < p < \infty$, and from the space M_{1,φ_1} to the weak space WM_{1,φ_2}. In the case $a \in BMO(\mathbb{R}^n)$, we find the sufficient conditions on the pair (φ_1, φ_2) which ensures the boundedness of the operators $[a, T]$ from M_{p,φ_1} to M_{p,φ_2}, $1 < p < \infty$, and from the space M_{1,φ_1} to the weak space WM_{1,φ_2}.

We also prove the boundedness of the sublinear operators T_α, $\alpha \in (0, n)$ satisfying condition (3) generated by Riesz potential operator from one generalized Morrey space M_{p,φ_1} to M_{q,φ_2}, $1 < p < q < \infty$, $1/p - 1/q = \alpha/n$, and from the space M_{1,φ_1} to the weak space WM_{q,φ_2}, $1 < q < \infty$, $1 - 1/q = \alpha/n$. In the case $a \in BMO(\mathbb{R}^n)$ and $[a, T_\alpha]$ be a sublinear operator, we find the sufficient conditions on the pair (φ_1, φ_2) which ensures the boundedness of the operators $[a, T_\alpha]$ from M_{p,φ_1} to M_{q,φ_2}, $1 < p < q < \infty$, $1/p - 1/q = \alpha/n$, and from the space $M_{L(1+log^+L),\varphi_1}$ to the weak space WM_{q,φ_2}, $1 < q < \infty$, $1 - 1/q = \alpha/n$.

RIEMANN-HILBERT PROBLEMS, LAX EQUATIONS AND SINGULARITIES

António Ferreira dos Santos

The application of Riemann-Hilbert problems to the study of Lax equations for finite-dimensional integrable systems is analysed. It is shown that the existence of a singularity of the solution at a point $t = t_i$ is directly related to the property that the kernel of a certain Toeplitz operator be non-trivial. An example of a dynamical system is presented.

SIMPLE NEUMANN EIGENVALUES FOR THE LAPLACE OPERATOR IN A DOMAIN WITH A SMALL HOLE. A FUNCTIONAL ANALYTIC APPROACH

Massimo Lanza de Cristoforis

Let $\mathbb{A}(\epsilon)$ be the annular domain obtained by removing from a bounded open domain $\mathbb{I}^o$ of $\mathbb{R}^n$ a small cavity of size $\epsilon > 0$. Then we assume that for some natural index l, $\lambda_l[\mathbb{I}^o] > 0$ is a simple Neumann eigenvalue of $-\Delta$ in $\mathbb{I}^o$, and we show that there exists a real valued real analytic function $\hat{\lambda}_l(\cdot, \cdot)$ defined in an open neighborhood of $(0, 0)$ in $\mathbb{R}^2$ such that the l-th Neumann eigenvalue $\lambda_l[\mathbb{A}(\epsilon)]$ of $-\Delta$ in $\mathbb{A}(\epsilon)$ equals $\hat{\lambda}_l(\epsilon, \kappa_n \epsilon \log \epsilon)$ and such that $\hat{\lambda}_l(0, 0) = \lambda_l[\mathbb{I}^o]$. Here $\kappa_n = 1$ if n is even and $\kappa_n = 0$ if n is odd. Thus in particular, we show that if n is even $\lambda_l[\mathbb{A}(\epsilon)]$ can be expanded into a convergent double series of powers of ϵ and $\epsilon \log \epsilon$ and that if n is odd $\lambda_l[\mathbb{A}(\epsilon)]$ can be expanded into a convergent series of powers of ϵ. Then related statements have been proved for corresponding eigenfunctions.

SOBOLEV INEQUALITIES FOR DIFFERENTIAL FORMS

Vladimir Goldshtein

We study relations between Sobolev inequalities for weakly differentiable differential forms on Riemannian manifolds (M, g) and the $L_{q,p}$-cohomology of these manifolds. The $L_{q,p}$-cohomology is defined to be the quotient of weakly closed differential forms of the class L_p modulo differential forms which are weak exterior differentials of differential forms in L_q. Roughly speaking an existence of the p, q-Sobolev inequality for differential forms is equivalent to vanishing of the corresponding $L_{q,p}$-cohomology. This observation will not lead us very far unless we are able to compute $L_{q,p}$-cohomology. Unfortunately, this is not an easy task but some computations for complete non compact manifolds and manifolds with singularities will be discussed.

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SOLUTION TO THE NAVIER-STOKES EQUATIONS WITH RANDOM INITIAL DATA

Evelina Shamarova

We construct a solution to the spatially periodic d-dimensional Navier–Stokes equations with a given distribution of the initial data. The solution takes values in the Sobolev space H^α, where the index $\alpha \in \mathbb{R}$ is fixed arbitrary. The distribution of the initial value is a Gaussian measure on H^α whose parameters depend on α. The Navier–Stokes solution is then a stochastic process verifying the Navier–Stokes equations almost surely. It is obtained as a limit in distribution of solutions to finite-dimensional ODEs which are Galerkin-type approximations for the Navier–Stokes equations. Moreover, the constructed Navier–Stokes solution $U(t, \omega)$ possesses the property:

$$\mathbb{E}[f(U(t,\omega))] = \int_{H^\alpha} f(e^{t\nu\Delta}u)\gamma(du),$$

where $f \in L_1(\gamma)$, $e^{t\Delta}$ is the heat semigroup, ν is the viscosity in the Navier–Stokes equations, and γ is distribution of the initial data.

This work generalizes, in several directions, the technique and the results of [1] where the authors prove the existence of the solution to the two-dimensional spatially periodic Euler equations. Unlike [1], our results hold for all viscosities $\nu \geqslant 0$ which includes the Euler ($\nu = 0$) and the Navier–Stokes ($\nu > 0$) cases. They hold for all dimensions $d \geqslant 2$, and for any Sobolev space index $\alpha \in \mathbb{R}$ whereas the result of [1] was proved for $\alpha < -\frac{1}{2}$.

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SOLVABILITY OF A DIRICHLET PROBLEM FOR A TIME FRACTIONAL DIFFUSION-WAVE EQUATION IN LIPSCHITZ DOMAINS

Jukka Kemppainen

In this talk we investigate a Dirichlet problem for a time fractional diffusion-wave equation (TFDWE)

$$\begin{aligned} \partial_t^\alpha \Phi - \Delta \Phi &= 0, \text{ in } Q_\infty = \Omega_\pm \times (0,\infty), \\ \Phi &= g, \text{ on } \Sigma_\infty = \Gamma \times (0,\infty), \qquad (4) \\ \Phi(x,0) &= \partial_t \Phi(x,0),\ x \in \Omega_\pm, \qquad (5) \end{aligned}$$

with $\Omega_\pm = \Omega_-$ or $\Omega_\pm = \Omega_+$, where $\Omega_- =: \Omega \subset \mathbb{R}^n$ is a bounded domain with Lipschitz continuous boundary Γ, $\Omega_+ = \mathbb{R}^n \setminus \overline{\Omega}$ is the exterior domain and

$$\partial_t^\alpha u(t) = \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-\tau)^{1-\alpha} \frac{\mathrm{d}^2 u}{\mathrm{d}\tau^2}(\tau)\,\mathrm{d}\tau \qquad (6)$$

is the fractional Caputo derivative of order $1 < \alpha < 2$.

We use the single layer approach to solve (4). This approach allows us to obtain a coercive and bounded sesquilinear form, which admits a unique solution. Therefore the problem (4) has a unique solution and the solution is given by the single layer potential.

SOLVING A CLASS OF TWO-DIMENSIONAL NONLINEAR VOLTERRA-FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

Shadan Sadigh Behzadi

In this paper, a two-dimensional nonlinear Volterra-Fredholm integro-differential equation is solved by using the modified Adomian decomposition method (MADM), variational iteration method (VIM), homotopy analysis method (HAM) and modified homotopy perturbation method (MHPM). The approximate solution of this equation is calculated in the form of series which its components are computed easily. The uniqueness of the solution and the convergence of the proposed methods are proved in detail. Numerical example is showed to demonstrate the accuracy of the present methods.

SOME NEW STEIN AND HARDY TYPE INEQUALITIES

Lars-Erik Persson

First we will discuss and present some ideas and results from my joint paper [1] with Stefan Samko. After that we will give an elementary proof of "all" Hardy inequalities for finite interval for the case with power weights and when we keep the best constant in all situations (see [2]). Finally, we will briefly describe some new results concerning characterizations of weighted Hardy type inequalities to hold. For example that the usual Muckenhoupt-Bradley, Mazya-Rosin and Persson-Stepanov conditions can be replaced by infinite many other alternative but equivalent conditions (see e.g. [3]). The results above can not be found in the standard literature, see e.g. the books [4]-[6] and the references given there.

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SOME RECENT DEVELOPMENTS OF SCATTERING PROBLEMS IN FLUID-SOLID INTERACTION

George Hsiao

We consider the interaction between an elastic body and a compressible inviscid fluid, which occupies the unbounded exterior domain. The problem of determining the shape of such an elastic scatter from the measured far-field pattern of the scattered fluid pressure field is of central importance in detecting and identifying submerged objects. The efficient numerical solution of the inverse problems of this type is challenging due to the fact that they are both nonlinear and severely ill-posed. In this lecture, we present two approaches recently developed based on nonlinear optimization and regularization. The first approach is a coupling procedure of field and boundary integral equations, and it requires the solutions of the direct problem for the numerical computation of minimizers. In the second approach, the acoustic and elastic waves are approximated by simple-layer potentials over auxiliary surfaces, and this method does not require a direct solver or an additional treatment of possible Jones modes. For illustration, some numerical experiments will also be included.

This lecture is based on joint works with J. Elschner and A. Rathsfeld of WIAS in Berlin.

SPECTRAL ANALYSIS OF ONE CLASS OPERATOR PENCIL WITH BARRIER POTENTIAL

Rakib Efendiev

We will investigate wave propagation in one-dimensional layered-inhomogeneous medium with barrier

$$-y'' + \Psi(x)y + V_0[\theta(x) - \theta(x-a)]y = \lambda^2\rho(x)y \tag{7}$$

where $\Psi(x)$ is a steplike potential of the form

$$\Psi(x) = \begin{cases} \sum\limits_{n=1}^{\infty} q_n^+ e^{inx} + \sum\limits_{n=1}^{\infty} 2\lambda p_n^+ e^{inx} & x < 0,\ x > a \\ \sum\limits_{n=1}^{\infty} q_n^- e^{inx} + \sum\limits_{n=1}^{\infty} 2\lambda p_n^+ e^{inx} & 0 < x < a \end{cases} \tag{8}$$

and $V_0[\theta(x) - \theta(x-a)]$ is a barrier potential with height V_0 and width a,

$$\theta(x) = \begin{cases} 0 & x < 0, \\ 1 & x \geq 0 \end{cases}$$

is a Heaviside step function and $\rho(x)$ has a form

$$\rho(x) = \begin{cases} 1 & for \quad x < 0, x > a \\ -\beta^2 & for \quad 0 < x < a \end{cases}$$

Without changing the results any other shifted is possible.

The barrier divides the spaces in three parts ($x < 0, 0 < x < a, x > a$) in any of these parts the potentials is complex, periodic meaning that layered-inhomogeneous medium can also absorption and emit an energy and wave propagation has different speed in each medium. The imaginary part of potential represents emission or absorbtion.

Our primary aim is to study the spectrum and solving the inverse problem for singular non-self-adjoint operator by normalizing numbers corresponding to quasi-eigenfunctions of the Sturm-Liouville operator with complex periodic potential and discontinuous coefficients on the axis.

SPECTRAL THEORY OF LARGE WIENER-HOPF OPERATORS WITH COMPLEX-SYMMETRIC KERNELS AND RATIONAL SYMBOL

Sergey Grudskiy

This report is devoted to the asymptotic behaviour of individual eigenvalues of truncated Wiener–Hopf integral operators over increasing intervals. The kernel of the operators is complex-symmetric and has a rational Fourier transform. Under additional hypotheses, the main result describes the location of the eigenvalues and provides their asymptotic expansions in terms of the reciprocal of the length of the truncation interval. Also determined is the structure of the eigenfunctions.

STABILIZATION OF TIMOSHENKO BEAMS

Nasser-eddine Tatar

The last three decades or so have witnessed a rapid development in high technologies using beams. This has stimulated a lot of interest and many results have been published. The Timoshenko bean theory is widely used to describe the dynamics of a beam when the transverse shear strain is significant. In contrast, the well-known Euler-Bernoulli theory does not take into account such an effect. In the derivation of Timoshenko equation it is assumed that the plane cross-sections remain plane but could be oblique to the centerline after deformation. Internal or external forces are transformed into mechanical vibrations. Often these vibrations are harmful to the structure and may result in malfunction of the structure. It is therefore of great importance to look for means and ways to suppress or at least reduce these vibrations. Several types of dampers have been designed for this purpose. In this paper we consider viscoelastic dampers provided by viscoelastic materials like rubber, some plastics and some glasses. These materials are capable of suppressing the vibrations. In practice the time of stabilization of the system is very important. It is preferable to have an exponential decay of the deflection to zero. This will depend on the nature of the viscoelastic material which is characterized by its relaxation function. Some classes of relaxation functions can be found in the literature. In this talk I will go over these classes and discuss the recent developments in this regard.

STRUCTURAL STABILITY OF NUMERICAL METHODS FOR BOUNDARY VALUE PROBLEMS OF FRACTIONAL DIFERENTIAL EQUATIONS

Maria Luísa Morgado

For a class of nonlinear fractional boundary value problems with order between 0 and 1, we present some results concerning the existence and uniqueness of the solution and we investigate the sensitivity of the solution with respect to the boundary condition. Some numerical methods are constructed and some numerical examples are presented and discussed.

This is a joint work with Neville Ford.

SYMBOLICAL AND FRACTIONAL INTEGRO-DIFFERENTIATION (CLASSICAL AND COMPUTER APPROACHES)

Oleg Marichev

There are known various generalizations of basic calculus operations (differentiation and integration) to the cases, where they are applied "symbolical n times" or "fractional order times", see the book (Samko S.G., Kilbas A.A., Marichev O.I. "Fractional Integrals and Derivatives (theory and applications)",

Gordon and Breach Science Publishers, 1993, 976 pp.) and publications of other authors.

For example, for non negative integer n the corresponding formula for n-th order derivative of sine is very simple and well known, see (http://functions.wolfram.com/01.06.20.0003.02 or http://www.wolframalpha.com/ and type SeriesCoefficient[Sin[x],{x,a,n}]), for arbitrary complex order c the corresponding formula for fractional integro-differentiation of power function z^a is also well known (http://functions.wolfram.com/01.02.20.0026.01).

The talk is devoted to comparison of the above mentioned "classical approach" to symbolical and fractional integro-differentiation with so called "computer approach", which we started to realize in computer system *Mathematica* on the base of huge collections of corresponding formulas in Wolfram Functions Site (see http://functions.wolfram.com/).

THE HARDY OPERATOR MINUS THE IDENTITY AND MINIMAL REARRANGEMENT INVARIANT SPACES

Javier Soria

We give a positive answer to a conjecture of Kruglyak and Setterqvist about the norm of the Hardy operator minus the identity on decreasing functions. This study leads us to consider a new class of minimal rearrangement invariant spaces, for which we also establish some functional properties.

THE OPERATOR $I\frac{D}{DX}$

Franciszek Szafraniec

The operator in question is a standard textbook example of an unbounded one, therefore everything concerning it seems to be definite. Nevertheless I intend to disclose the (still) hidden part of its live. The reference is D. Cichoń, J. Stochel and F.H. Szafraniec, Naimark extensions for indeterminacy in the moment problem. An example, *Indiana University Mathematics Journal*, to appear.

THE ROLE OF S.G.SAMKO IN THE ESTABLISHING AND DEVELOPMENT OF THE THEORY OF FRACTIONAL DIFFERENTIAL EQUATIONS

Sergei Rogosin

The report is devoted to the description of results by Professor Stefan Samko in different directions of modern analysis, namely:

- solvability and closed form solution to singular integral equations;
- solvability and closed form solution to integral equation with weak singularities;
- investigation of the convolution type integral equations;
- study of analytic, functional, composition and asymptotic properties of integral operators (in particular, one- and multi-dimensional potential type operators);
- investigation of properties of hypersingular operators;
- different aspects of fractional calculus;
- introducing and study of fractional powers of operators;
- introducing and study of functional spaces with variable exponents and operators in these spaces.

Main attention will be paid to those results which have the most essential influence on the creation and development of the modern theory of fractional differential equations.

It will be also discussed the role of the so-called "Bible of Modern Fractional Calculus" [1-2].

Acknowledgement. The work is partially supported by Belarusian Fund for Fundamental Scientific Research, grant F10MS-024.

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TOPOLOGY CHANGING FOR MINIMIZERS OF MONGE-KANTOROVICH PROBLEM WITH AVERAGE DISTANCE ENERGY FUNCTIONAL

Xin Yang Lu

Given a locally convex domain Ω in R^2 with finite Hausdorff measure boundary, and an "energy" functional F, we will consider the p-Laplacian problem, i.e.

$$\begin{cases} -\Delta_p u(x) = f(x) \ in \ \Omega\backslash\Sigma \ (\Delta_p \text{ denotes the } p\text{-Laplacian}) \\ u(x) = 0 \ on \ \Sigma \end{cases}$$

where f is a preassigned function. Then we fix the length l for Σ and we seek for those Σ whose solution u of the previous system minimizes $F(u)$, i.e. the solutions u such that

$$F(u) = \min_v F(v)$$

with v solution of the previous system (Σ is varying among all Hausdorf one-dimensional connected sets with measure l).

If we let p go to infinite, the p-Laplacian problem converge (in the sense of convergence of operators) to the Monge-Kantorovich problem with free Dirichlet regions. Considering the minimizing movement problem (both quasi-static and dynamic case), we will analyze the topological proprieties of minimizers in the evolution with emphasis on when it changes topology, under irreversibility assumption. We will explicitly compute the "branching time" (time t in the evolution at which the minimizing set Σ_t changes topology) upperbounds in the rate-independent case, showing that even very similar initial conditions can lead to totally different behaviors.

Moreover, we will show that significant difference exists between the quasi-static and dynamic case, with the latter situation presenting strong differences as the dissipation exponent is above or below $3/2$, as with exponent above $3/2$ the evolution continues indefinitely, but otherwise it can halt definitively after some time.

UPPER BOUNDS FOR SCHRODINGERIAN SUBHARMONIC FUNCTIONS ADMITTING CERTAIN LOWER BOUNDS

Alexander Kheyfits

We extend Matsaev's upper bounds for the classical harmonic functions admitting certain lower bounds onto subsolutions of some stationary Schrodinger equations

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VARIABLE EXPONENT SMIRNOV CLASSES AND BVP IN NON-STANDARD BANACH FUNCTION SPACES

Vakhtang Kokilashvili

In our lecture we plan to present the results concerning new classes of analytic and harmonic functions and boundary value problems in (single and doubly connected) domains with nonsmooth boundaries within the framework of some nonstandard Banach function spaces.

The following tasks will be discussed:

i) to introduce and study the properties of various type Hardy and Smirnov weighted classes of analytic and harmonic functions when integrability exponent is a function; to explore the problem of representability by the Cauchy type integrals.

ii) to reveal the influence of the recent results on mapping properties of Cauchy singular integral operator on nonstandard Banach function spaces in the theory of BVPs for analytic and harmonic functions.

iii) to solve the Dirichlet problem for harmonic functions of variable exponent Smirnov classes in (single and doubly connected) domains with piecewise smooth boundaries; to reveal the influence of the geometry of the boundary on the solvability picture; to construct solutions in quadratures.

iv) to give the solution of the Riemann problem of linear conjugation when the right-hand side of the boundary condition belongs to the Grand Lebesgue space.

The presented results are based on the joint research with V.Paatashvili.

VOLTERRA-WIENER SERIES SOLUTION FOR NONLINEAR INFINITE DIMENSIONAL VOLTERRA-DUSHNIK INTEGRAL EQUATIONS IN THE SPACE OF THE REGULATED FUNCTIONS

Luciano Barbanti

This paper deals with a second order Volterra –Stieltjes nonlinear integral equation in which the integral considered is the Dushnik one on regulated functions in general Banach spaces. These equations enclose a very large number of dynamical systems, as the Ordinary Differential equations, the delayed and Partial Differential Equations among others. The main results in the work are the Riesz kind representation theorems for multidimensional operators that appear naturally in the problem by extending Morse –Transue results for the continuous case.

This is a joint work with Marcia Federson, Berenice Camargo Damasceno, Fabio Roberto Chavarette.

WAVE FACTORIZATION: BEFORE AND BEYOND

Vladimir Vasilyev

One considers the concept of wave factorization in a context of general theory of boundary value problems for the pseudo differential operators, its relation with classical theory of Riemann problem. One discusses a multivariable linear conjugation problem of V.S. Vladimirov and its connections with others multivariable variants. It's shown it makes a sense to study a discrete theory also, which leads to periodical analogues of stated results. Limit transfer from discrete theory to continuous ones for some special cases is justified, it is based on interesting spectral properties of Calderon-Zygmund operators.

WEIGHTED MODULAR AND NORM INEQUALITIES FOR THE HARDY OPERATOR IN VARIABLE L^P SPACES OF MONOTONE FUNCTIONS

Santiago Boza

We study weak and strong type modular inequalities on weighted $L^{p(\cdot)}$ spaces with a variable exponent $p(\cdot)$ for the Hardy operator restricted to non-increasing functions. We show that the exponents $p(\cdot)$ for which these modular inequalities hold must have a constant oscillation at zero, which implies that these exponents are either constant or extremely oscillating near the origin. For this purpose, and similarly to the constant case, we introduced the class of weights $B_{p(\cdot)}$. We also deal with the problem of the boundedness in norm in the same context.

The talk is part of a joint work with Javier Soria (University of Barcelona).

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